Solutions to Four Sample Questions

1. Answer: D.

Thirty-five consecutive integers can be expressed as $n$, $n + 1$, $n + 2$, ..., $n + 34$. The sum of the first 10 is $10n + 45$. The sum of the next 25 is $25n + 550$. The difference between these two quantities is $15n + 505$. Set this equal to 1900 and solve for $n$. We get $n = 93$. The largest of the numbers is 93 + 34 = 127.

2. Answer: B.

A number that is divisible by 9 has the property that the sum of the digits is divisible by 9. Consider the repeating digits in order. 00, 11, 22, 33, 44, 55, 66, 77, 88, 99. The third digit that is included in the number is uniquely determined. In the case of 00, the third digit must be a 9. The only possibility is 900. In the case of 11, the third digit must be a 7. So, there are three possibilities, 117, 171, and, 711. Likewise there are three cases for three digit numbers that include 2-2-5, 4-4-1, 5-5-8, 7-7-4, and 8-8-2. In the case of 9-9-0, there are 2 possibilities, 990, and 909. We do not count the case of 3-3-3 and 6-6-6 since neither of these has exactly 2 repeating digits. So, the answer is 1+3+3+3+3+3+3+2 = 21

3. Answer: E

When connecting two consecutive midpoints, an isosceles triangle is formed having a vertex angle of measure $120^\circ$ and legs of measure 2 cm. Drawing the angle bisector of the vertex angle gives two 30-60-90° triangles with the result that the segment joining the midpoints is of length $2\sqrt{3}$. The area of the original hexagon, using the formula $\frac{3}{2}s^2\sqrt{3}$ is $24\sqrt{3}$ sq. cm. The area of the second hexagon is $18\sqrt{3}$ sq. cm. The ratio of the area of a hexagon to the next larger one is $\frac{3}{4}$. The areas of this collection of hexagons form an infinite geometric progression. The total area is $\frac{a}{1-r} = \frac{24\sqrt{3}}{1-\frac{3}{4}} = 96\sqrt{3}$ sq. cm.

4. Answer: A.

Factor all of the trinomials. $\left((x+1)(x+3)\right)^{(x-3)(x-1)} = \left((x-4)(x+1)\right)^{(x-1)(x+4)}$. If $x = -1$, we get $0^8 = 0^{-6}$, but the right side is $0^{-6}$, which is undefined. If $x = 1$, we get $8^0 = (-6)^0 \Rightarrow 1 = 1$, thus the sum of the rational solutions is 1.